‘AS’ - symbolic analysis program

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AS ver 4.D
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Disclaimer

AS is a free program and you may use it at your own risk.

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0. Revisions

Date	Rev	Description
09.IV.2014	4_1	A public version for AS_4D

1. Introduction

1.1. Features.

Program ‘**AS**’ performs a symbolic analysis of the linear circuits models. Symbolic analysis means, that all results of analysis are represented by symbols rather than the numbers. ‘**AS**’ is able to compute transfer functions, it’s sensitivities or admittance matrix co-factors. In the computations **Y**-matrix techniques are used. Program uses input description files and produces the results in the text format.

The program can be useful in the analysis of small circuits, which model only the core properties. There is a theory, which says that the usefulness of equation falls to zero, when it is longer then 5cm. Common sense is required, because **AS** can easily produce much longer results.

1.2. Circuit description.

Circuit description is similar to standard **Spice** syntax, but with some exceptions (see current source)
Text circuit specification requires sequential upward node numbering from 0 for the ground. **Rimu** schematics [1] can easily be adopted to automate this task. Program accepts circuit model which is build using the following components:

V - voltage source
I - current source
A - ideal operational amplifier
G - voltage controlled current source
F - current controlled current source
E - voltage controlled voltage source
H - current controlled voltage source
Y - admittance
C - capacitor
L - inductor
R - resistor (available, however usage of admitance Y=1/R is recommended)
Z -impedance (available, however usage of admitance Y=1/Z is recommended)
PU – probe - voltmeter
PI – probe - ammeter
PZ – probe – ohmmeter

Parts are identified by the first letter except for probes, which use two letters for identifications.

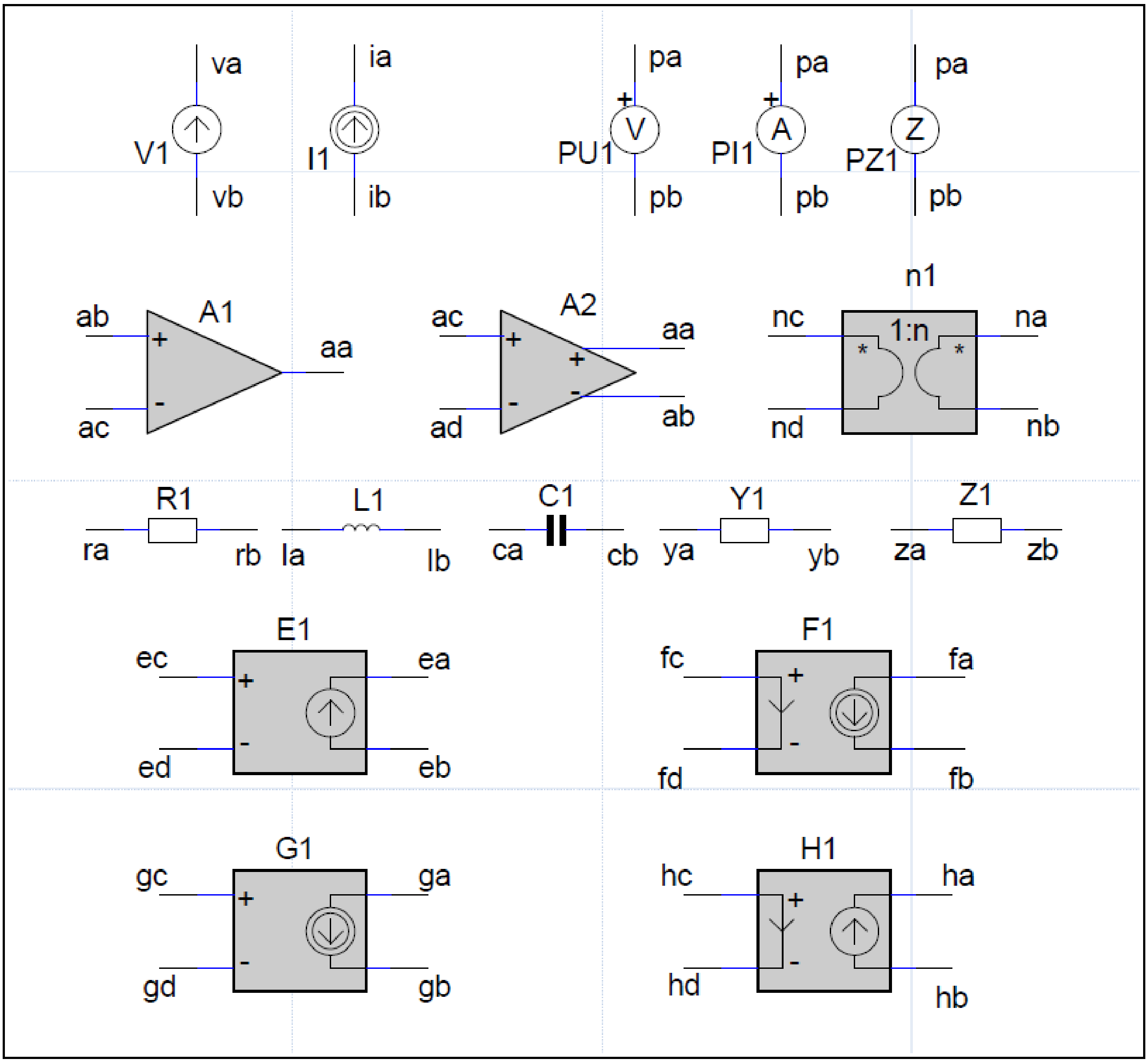


Fig. 1. All parts in Rimu schematics

V - Voltage Source
    Vxxx va vb

I - Current Source
    Ixxx ia ib

P – Probes (voltage, current, impedance)
    PUxx pa pb
    PIxx pa pb
    PZxx pa pb

A – Amp (Ideal):
    Axxx aa ab ac
    Axxx aa ab ac ad

E – VCVS:
    Exxx ea eb ec ed

F – CCCS:
    Fxxx fa fb fc fd

G - VCCS:
    Gxxx ga gb gc gd

H – CCVS:
    Hxxx ha hb hc hd

N – Transformer Ideal :
    Nxxx na nb nc nd

R – Resistor
    Rxxx ra rb

C – Capacitor:
    Cxxx ca cb

L – Inductor:
    Lxxx la lb

Y - admitance
    Yname ya yb

Z - impedance
    Zname za zb

NOTE 1:
R,Z are available, but using admittances Y = 1/R (1/Z) is recommended. Expression with admittances have more concise form because are natural for a circuit analysis using admittance matrix description. It is easy to substitute admittances with resistances or impedances later in a computer algebra system, like Maxima.

WARNING:
Current flows from ib to ia - ia node is a source (not sink like in **SPICE**). Thus, this convention is different than in **SPICE**.

2. Examples.

Maxima [2] is a system for the manipulation of symbolic and numerical expressions thus, symbolic analysis performed by '**AS**' makes it a natural companion of **Maxima**.
The examples show how '**AS**' symbolic expressions may be processed by **Maxima**.

NOTE1 :
To comply with **Maxima** syntax, after copying expressions from **AS** to Maxima, all equal signs (=) have to be replaced by comma (**:**). To prevent **Maxima** from showing **AS** expressions, ending semicolons (;) might be replaced by dollar signs ($).

NOTE2 :
Depending on target enviroment for the **Maxima** script, examples have different extensions :

- *.**mac** files are xmaxima batch files.
- ***.wxm** are wxMaxima documents
- ***.wxmx** are wxMaxima 'xml' documents

NOTE3 :
In examples following marking pattern is used.

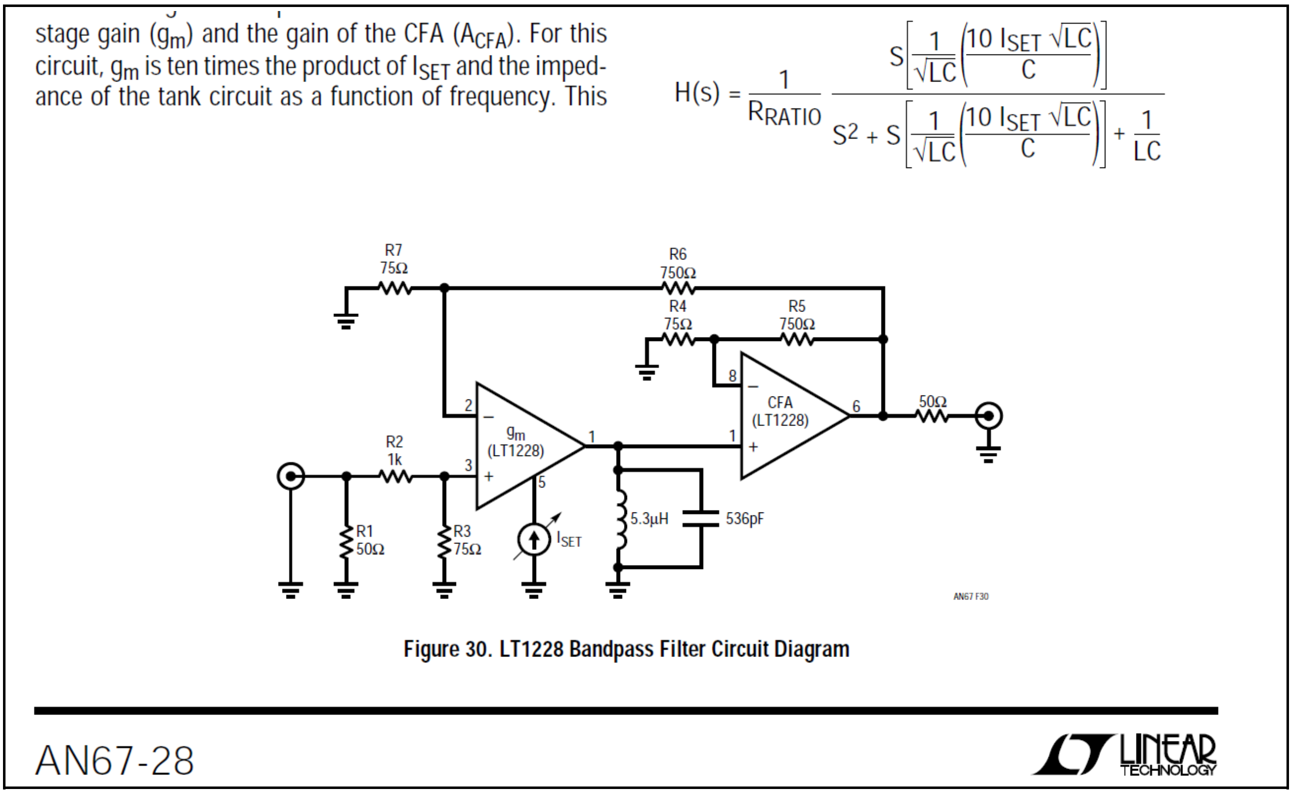
Expressions copied from AS have blue boarders.

Expressions copied from MAXIMA have red boarders.

Expressions copied from ELSEWHERE have green boarders.

2.1. Tuned band-pass filter

The original design of tuned band pass filter was presented in application note 67 by Linear Technology [1].


stage gain (g_m) and the gain of the CFA (A_{CFA}). For this circuit, g_m is ten times the product of I_{SET} and the impedance of the tank circuit as a function of frequency. This

$$H(s) = \frac{1}{R_{RATIO}} \frac{S\left[\frac{1}{\sqrt{LC}}\left(\frac{10\, I_{SET}\sqrt{LC}}{C}\right)\right]}{S^2 + S\left[\frac{1}{\sqrt{LC}}\left(\frac{10\, I_{SET}\sqrt{LC}}{C}\right)\right] + \frac{1}{LC}}$$

Figure 30. LT1228 Bandpass Filter Circuit Diagram

AN67-28

Fig. 2.1. Original application note with tuned band-pass filter.

Simplified model omits input divider and output matching resistance, since voltage gain **K=U1/V1,** for equivalent circuit is calculated.

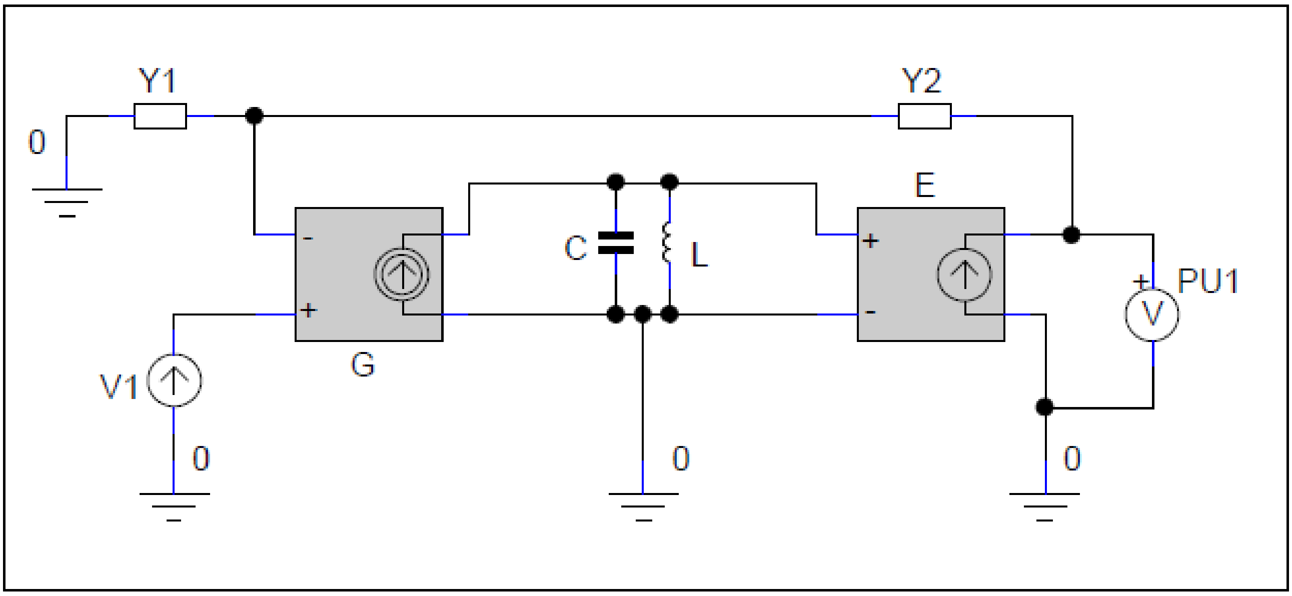


Fig. 2.2. Simplified model of gain stage of tuned band-pass filter.

Voltage gain **K** corresponds to controlled voltage source (**Vo=K*Vi**) in an equivalent Thevenin's model of the amplifier.

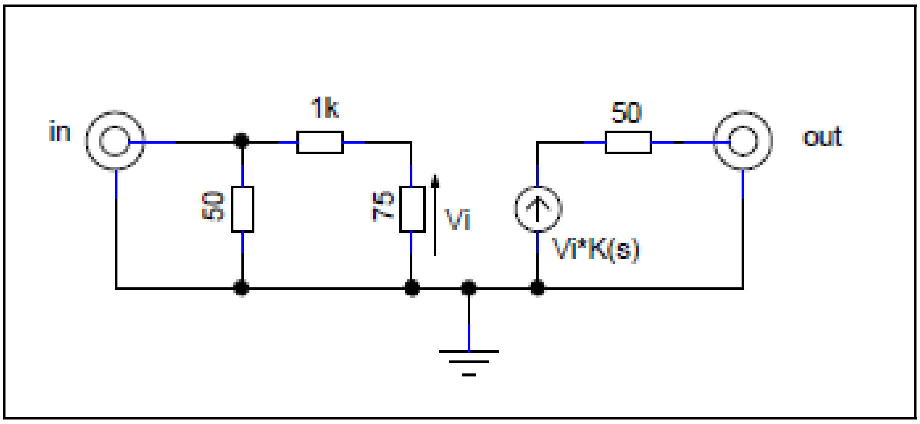


Fig. 2.3. Voltage gain K(s) in the equivalent model of the filter.

Voltage gain **K** is calculated in the form of expressions:

```
//  U1=K*V1
Da= (gi)*(s*(C*Y1+C*Y2)+Y2*G*E+s^(-1)*(Y2*(1/L)+Y1*(1/L)));

Db= (gi)*(Y1*G*E+Y2*G*E);
K=Db/Da;
```

Expressions from AS are simplified and reduced by Maxima:

$$K = \frac{s\,(E\;G\;L\;Y2 + E\;G\;L\;Y1)}{s^2\,(C\;L\;Y2 + C\;L\;Y1) + s\;E\;G\;L\;Y2 + Y2 + Y1} \quad [2.1]$$

K has a similar form as a general form of a second order band pass filter [2.2]

$$K(s) = K0\,\frac{s\,\frac{\Omega}{Q}}{s^2 + s\,\frac{\Omega}{Q} + \Omega^2} \quad [2.2]$$

Gain stage parameters:

Ω center frequency,
Q quality factor
K0 gain at frequency Ω

are calculated as :

$$\left[\,K0 = \frac{Y2 + Y1}{Y2},\; \Omega = \frac{1}{\sqrt{C}\,\sqrt{L}},\; Q = \frac{\sqrt{C}\,(Y2 + Y1)}{E\;G\,\sqrt{L}\;Y2}\,\right] \quad [2.3]$$

2.2. Current loop transmitter

One of many versions of current loop transmitter is shown below [Fig **2.4**]. This design was presented in **Maxim** application note [3].

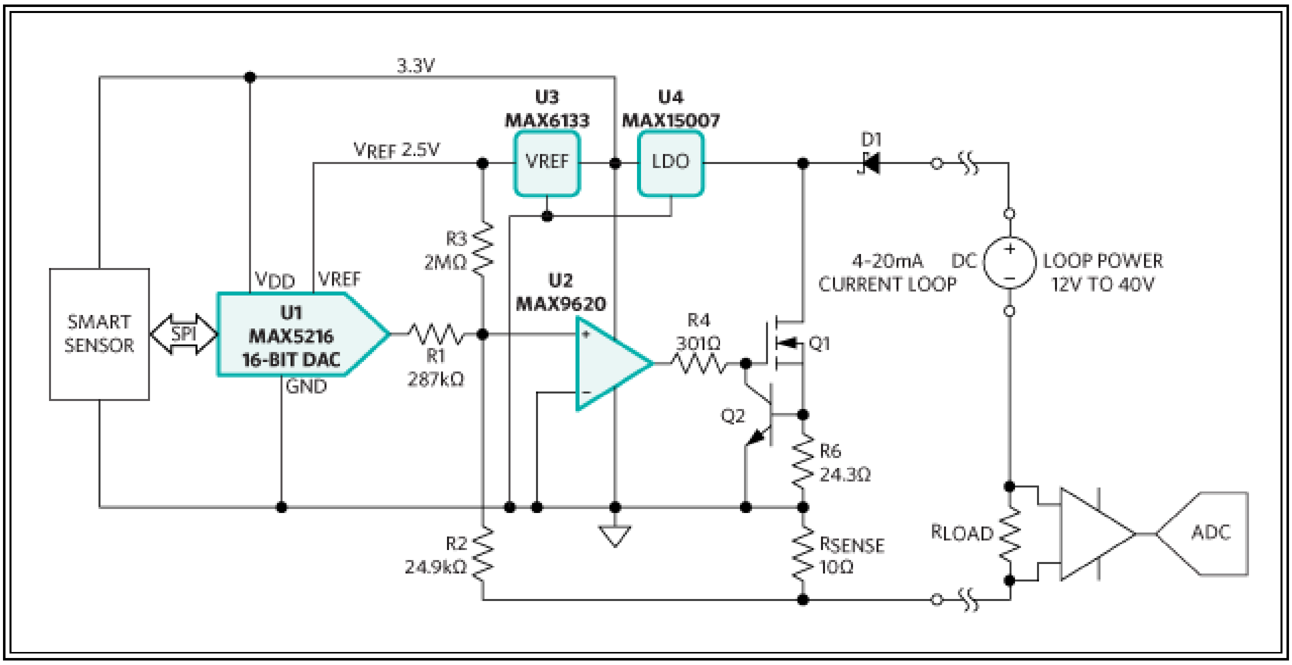


Fig. 2.4. Original schematics of current loop transmitter.

The linearized model of the design used in AS simulation is shown in next figure [Fig. **2.5**].
In the model the transmitter is powered by voltage source **Ve**, the output current **I** is measured by current probe **PI**. The internal reference is modeled by voltage source **Vr.** DAC's output voltage source is represented by **Vd**. Current source **Idd** models power supply of **[U1..U4]**. The operational amplifier **A1** is ideal. A MOS transistor is represented by DC small signal model – i.e. by transconductance **gm** and output admittance **yds**. Bipolar transistor is not open for normal operation and modeled as open circuit.

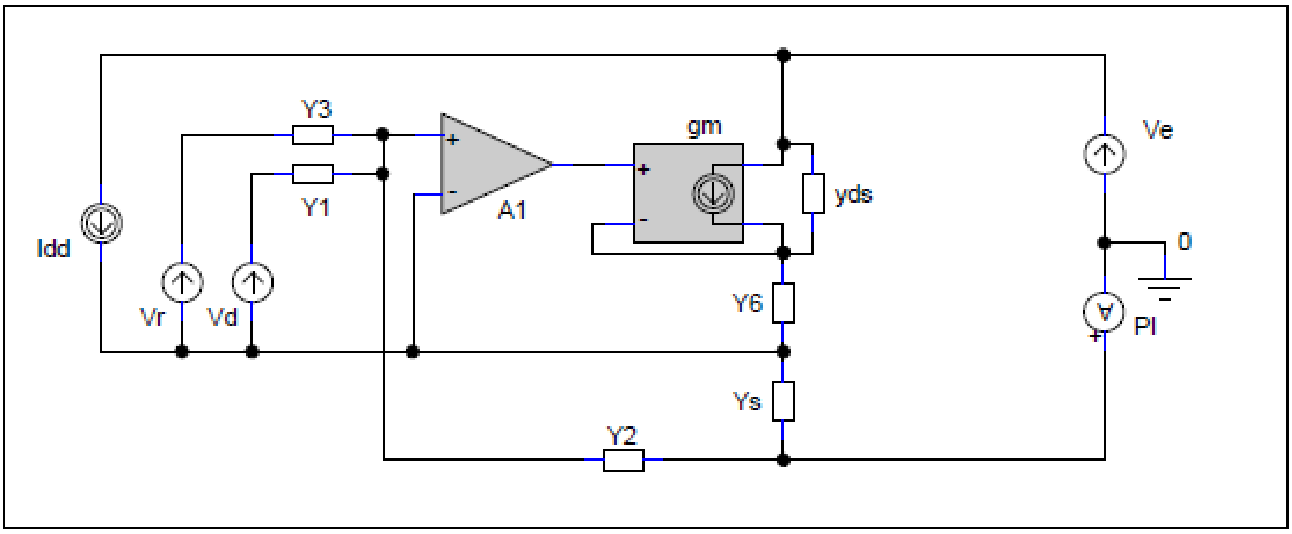


Fig. 2.5. Simplified linear model of current loop transmitter.

The output current **I** is calculated as a set of expressions:

```
//...   TF  ...
//  I=Nd*Vd+Ne*Ve+Nr*Vr+KIdd*Idd
da= (gm)*(Y6*Y2);

dbNd= (gm)*(Ys*Y6*Y1+Y6*Y2*Y1);
Nd=dbNd/da;
dbNe=0;
Ne=dbNe/da;
dbNr= (gm)*(Y6*Y3*Y2+Ys*Y6*Y3);
Nr=dbNr/da;
dbKIdd=0;
KIdd=dbKIdd/da;
```

Maxima calculates **I** as a function of **Vr**,**Vd** and circuit parameters. Original expression for current **I** might be transformed by substituting admittances **Y** by resistances **R** and redefining expression **I** as a function **I(Vr,Vd) [2.4]**.

$$I=\frac{(Vr\ Y2+Vr\ Ys)\ Y3+Vd\ Y1\ Y2+Vd\ Ys\ Y1}{Y2}$$

$$I=\frac{(Vd\ R2+Rs\ Vd)\ R3+Vr\ R1\ R2+Rs\ Vr\ R1}{Rs\ R1\ R3}$$

$$I(Vr,Vd)\ =\ \frac{Vr\ R2}{Rs\ R3}+\frac{Vr}{R3}+\frac{Vd\ R2}{Rs\ R1}+\frac{Vd}{R1} \qquad [2.4]$$

MOS is modeled by linear small signal model and thus, all currents and voltages should be considered as small signal variations around point of operation. When **Idd** is small enough and **V3** is large enough for linear operation of **A1** amplifier, current **I** depends only on voltage sources **Vr**,**Vd.** Therefore, the transmitter operates as voltage controlled current source, as long as **A1** operates linearly.

The actual design is supposed to be calibrated by setting zero and full scale voltages for a **DAC**.
To make a basic check of the design, we can calculate values of Vd corresponding to zero (**4mA**) and full scale (**20mA**):

$R1[kOhm]\ =\ 287.0$

$R2[kOhm]\ =\ 24.9$

$R3[kOhm]\ =\ 2000.$

$Rs[Ohm]\ =\ 10.0$

$I\_mA(Vr,Vd)\ =\ 1.245\ Vr+8.679\ Vd$

$Vref\ =\ 2.5$

$Vd0\ =\ 0.1\quad I\_mA(Vref,Vd0)\ =\ 4.0$

$Vd1\ =\ 1.946\quad I\_mA(Vref,Vd1)\ =\ 20.0$ [2.5]

For nominal values of resistors and reference source **Vr**, DAC output voltage for zero (**4mA**) is about **Vd=0.1V,** and for full scale (**20mA**) **Vd=1.95V**, which seems well within linear region of operation for amplifier and **DAC** with **Vr=2.5V** reference.

2.3. Photo-diode I/V converter – stability analysis.

There is a study of the problem in a book by **Analog Devices** [4].

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Compensation of a High Speed Photodiode I/V Converter

A classical I/V converter is shown in Figure 4-54 below. Note that it is the same as the previous photodiode preamplifier, if we assume that R1 >> R2. The total input capacitance, C1, is the sum of the diode capacitance and the op amp input capacitance. Dynamically, this is a classical second-order system, and the following guidelines can be applied in order to determine the proper compensation.

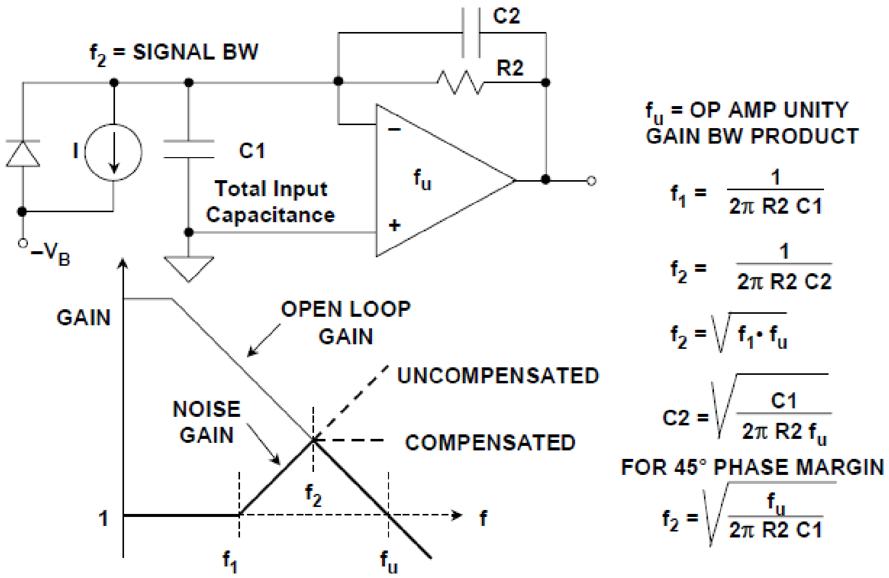

Fig. 2.6. Excerpt from a AD book on the compensation of photo-diode converter.

This simple circuit has a tendency to oscillate due to the phase shift in a feedback loop created by relatively large feedback resistance **R2** and photo-diode capacitance **C1** (including capacitance of op-amp input).
Feedback resistance **R2** is determined by required sensitivity of the current to voltage conversion.

$$R2=\frac{\Delta U}{\Delta I} \qquad [2.6]$$

Photo-diode capacitance **C1** is device dependent. For a given sensitivity **R2**, and photo-diode capacitance **C2**, value of compensation capacitance **C2** depends on amplifier gain-bandwidth product $\mathbf{f_t}$ (in figure 2.6 **GBW** is unconventionally denoted as $\mathbf{f_u}$)

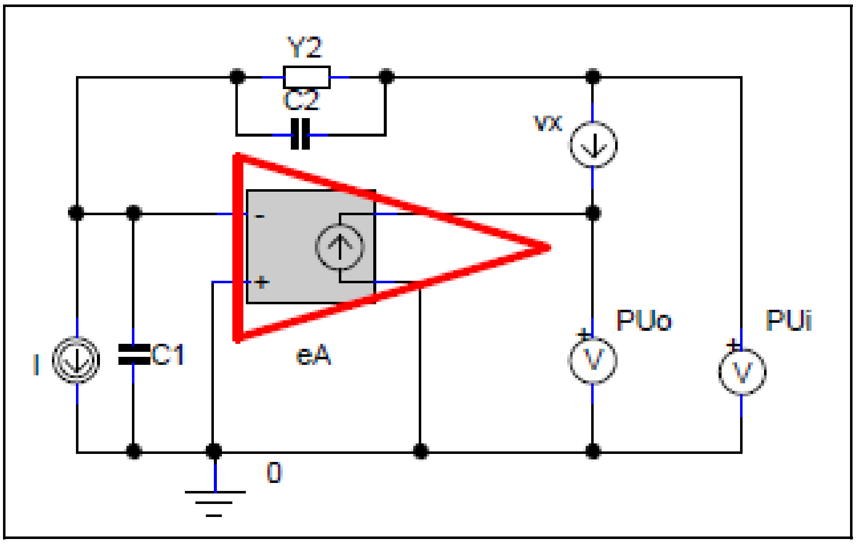

Fig. 2.7. Model of photo-diode converter for stability analysis.

For a linear model shown in figure 2.7 **AS** calculated expressions:

```
//...   TF  ...
//  Ui=Kix*vx+Mi*I
Da= (gi)*(s*(C2*eA+C1+C2)+Y2*eA+Y2);

DbKix=-(gi)*(s*(C1+C2)+Y2);
Kix=DbKix/Da;
DbMi= (gi)*(eA);
Mi=DbMi/Da;
//...   TF  ...
//  Uo=Kox*vx+Mo*I
DbKox= (gi)*(s*(C2*eA)+Y2*eA);
Kox=DbKox/Da;
DbMo= (gi)*(eA);
Mo=DbMo/Da;
```

The transfer function from **I** (photo-diode current) to **Uo** (output voltage) is denoted as **Mo=Uo/I | (vx=0)**. Additional voltage source **vx** replicates the open-loop measuring technique. With this approach, open loop gain T(s) gain is determined by the ratio of two voltages Uo, Ui, both being responses to injected voltage **vx** only (for I=0).

$$T(s) = -\frac{Uo}{Ui}\bigg|_{I=0} = b(s)\cdot A(s)$$

where b(s) is feedback gain.

In AS model op-amp is modeled as VCVS **eA**. Later, in Maxima VCVS gain eA is replaced by transfer function [2.7]

$$eA = A(s) = \frac{s}{2\pi f_t}\,\frac{\sqrt{2}}{1+\dfrac{s}{2\pi f_t}} \qquad [2.7]$$

With this approximation operational amplifier has a gain-bandwidth product $\mathbf{f_t}$, but due to second pole the phase margin is reduced to 45 degrees.

Application note by Analog Devices provides design equation to calculate value for **C2** for phase margin of 45 degrees. This equation are shown in figure 2.6.

For given parameters of the circuit,

```
ft  [MHz]=  16.0
R2  [kOhm]=  100.0
C1  [pF]=  5.0
```

[2.8]

```
C2[pF]  =  0.71
f1[MHz]  =  0.32
f2[MHz]  =  2.257
```

[2.9]

Design equations give compensation capacitance **C2=0.7pF,** which should provide a phase margin around **45** degrees for a cross-over frequency about **2.2MHz**.

Moreover, according to the text, doubling compensation capacitance should provide a design with phase margin around **60** degrees.

Verification of the designs by **Maxima**, proves design equations quite accurate and useful.

Results of analysis for **C2=0.7pF**, **C2=1.4pF C2≈0.0pF** are shown below.

For compensation **C2=0.7pF**, phase margin is **49** degrees, cross over frequency is **fx = 3.3 [MHz],**

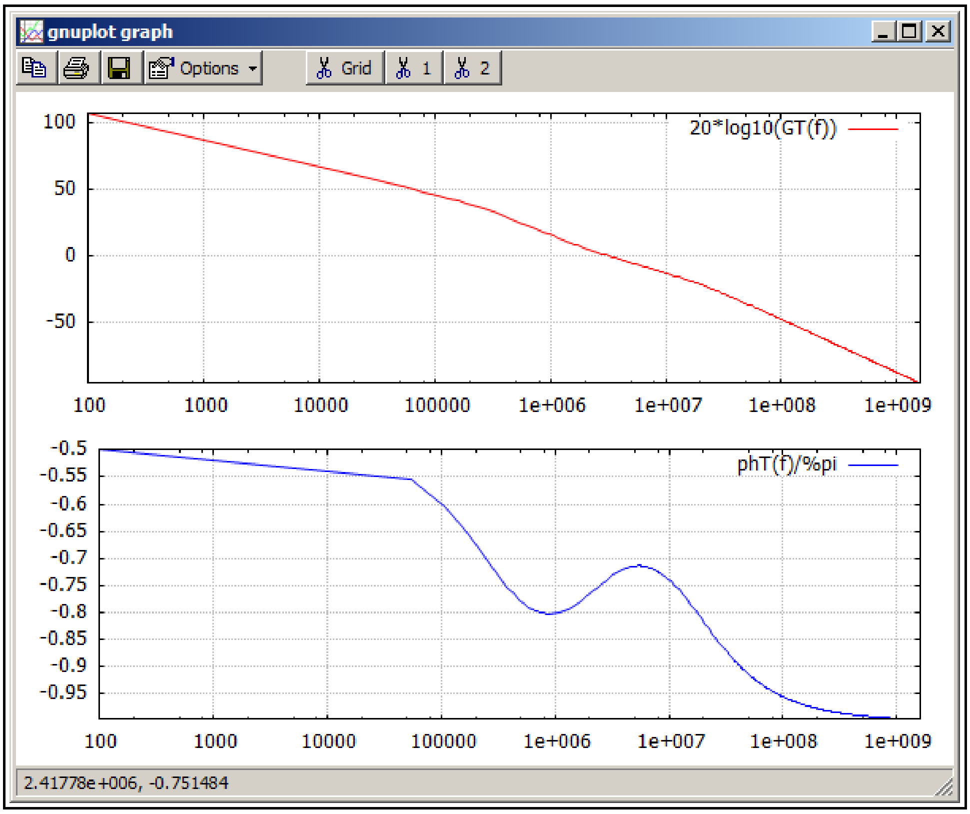

Fig. 2.8. Open loop gain for compensation capacitor **C2=0.7pF**.

and small signal (**I=1uA**) step response has small overshot.

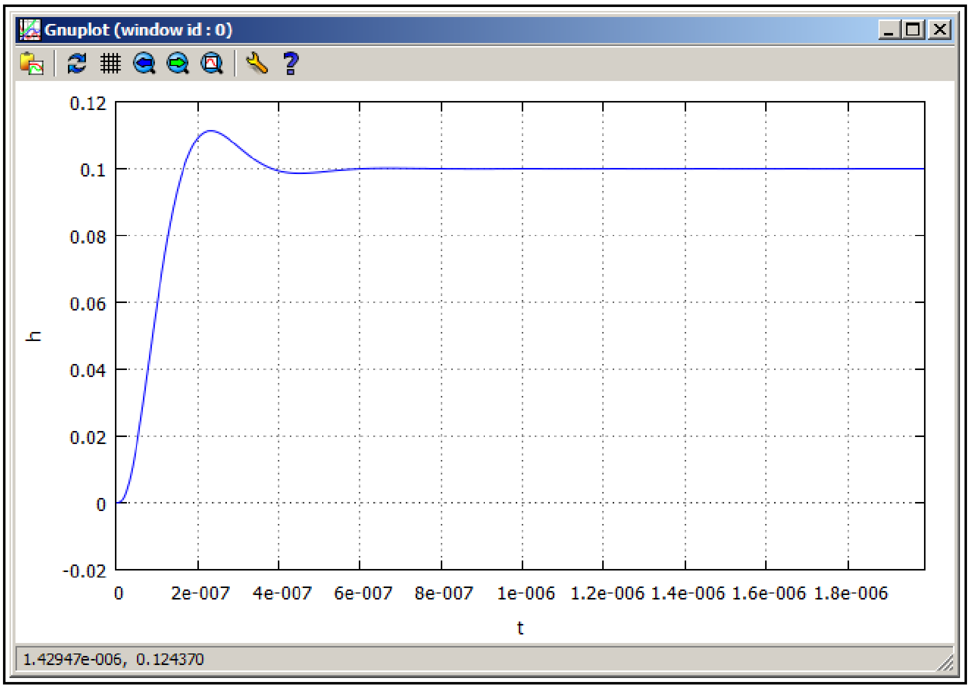

Fig. 2.9. I= 1uA step response gain for compensation capacitor **C2=0.7pF**.

Doubling compensation capacitance to **C2 =1.4pF** gives phase margin of 63 degrees, crossover frequency fx = 4.8[MHz]

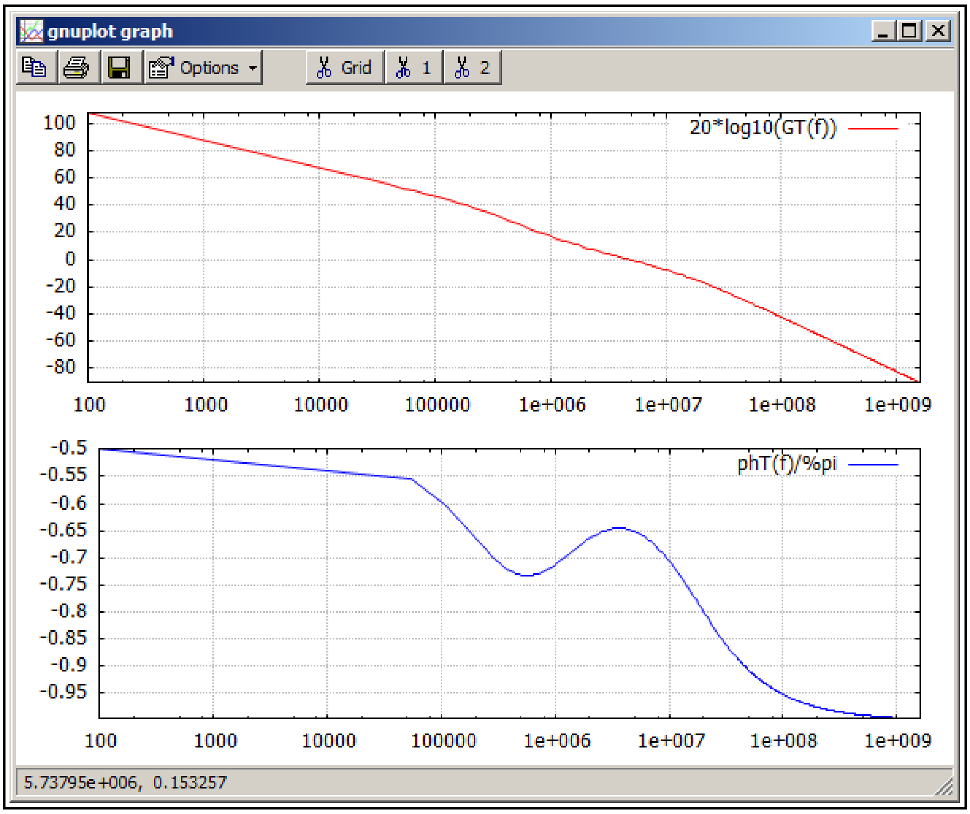

Fig. 2.10. Open loop gain for compensation capacitor **C2=1.4pF**.

and small signal (**I=1uA**) step response without overshot.

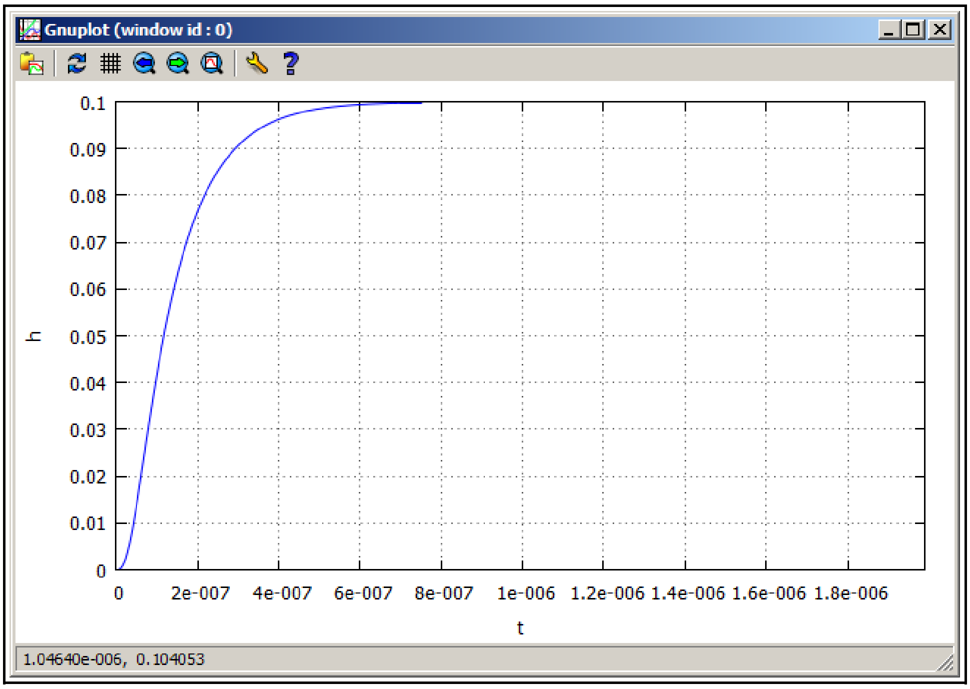

Fig. 2.12. I= 1uA step response gain for compensation capacitor **C2=1.4pF**

Negligible value of **C2 = 0.1pF**, gives very small phase_margin = 6.769, with cross-over fx [MHz]= 2.648,

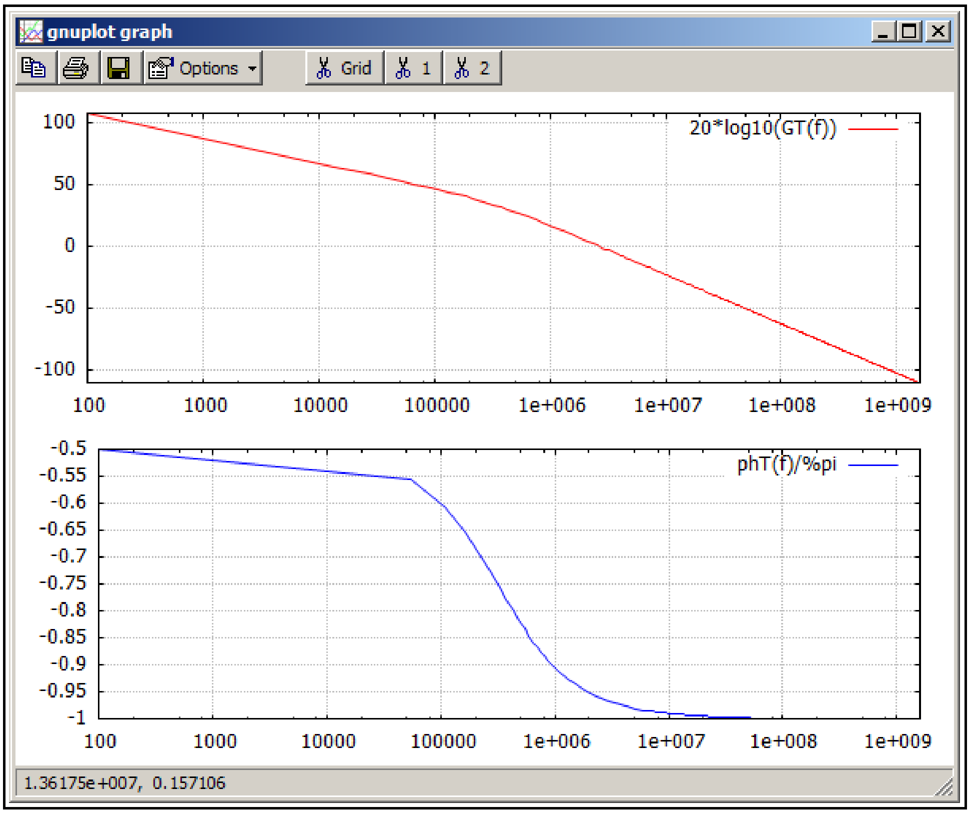

Fig. 2.13. Open loop gain for compensation capacitor **C2=0.1pF**.

and small signal (**I=1uA**) step response with a huge tendency for oscillation.

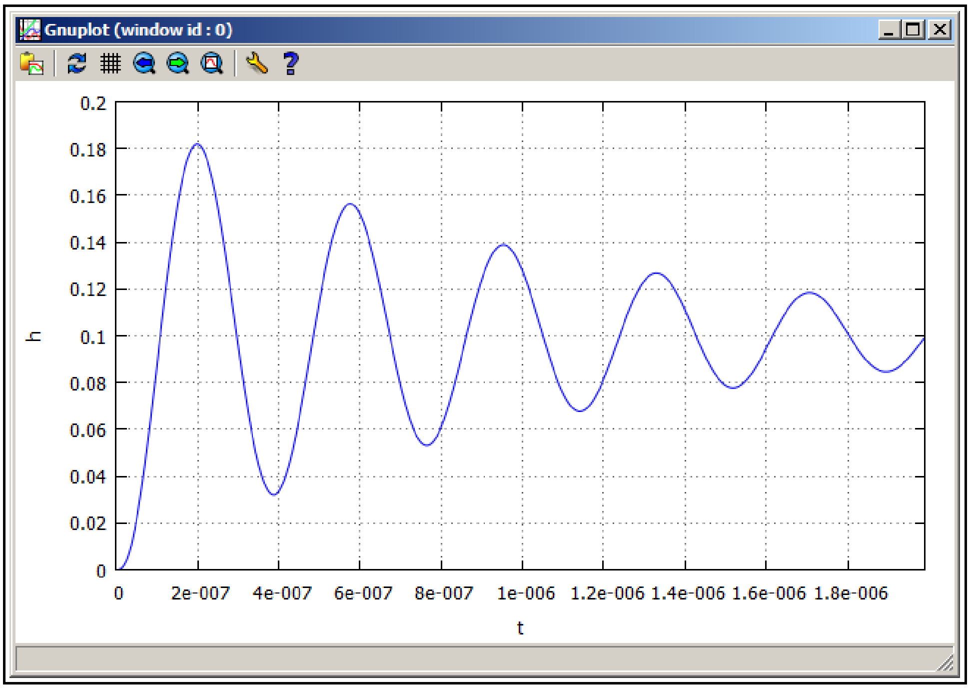

Fig. 2.14. I= 1uA step response gain for compensation capacitor **C2=0.1pF**

2.4. Photo-diode I/V converter – noise analysis.

A design discussed in previous section is used for an example noise analysis. Again we use a study of the problem in a book by **Analog Devices** [4] as a reference.

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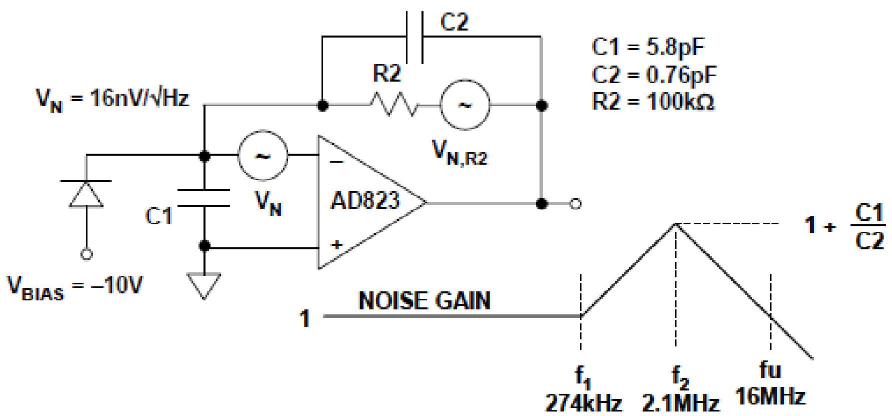
High Speed Photodiode Preamp Noise Analysis

As in most noise analyses, only the key contributors need be identified. Because the noise sources combine in an RSS manner, any single noise source that is at least three or four times as large as any of the others will dominate.

In the case of the wideband photodiode preamp, the dominant sources of output noise are the input voltage noise of the op amp, V_N, and the resistor noise due to R2, $V_{N,R2}$ (see Figure 4-58). The input current noise of the FET-input op amp is negligible. The shot noise of the photodiode (caused by the reverse bias) is negligible because of the filtering effect of the shunt capacitance C1.



- V_N RTO NOISE $\approx V_N\left[1+\frac{C1}{C2}\right]\sqrt{1.57\,f_2}$ = 250μV RMS
- $V_{N,R2}$ RTO NOISE $\approx \sqrt{4kTR2 \cdot 1.57 f_2}$ = 73μV RMS
- TOTAL RTO NOISE $= \sqrt{250^2 + 73^2}$ = 260μV RMS
- DYNAMIC RANGE $= 20\log\left[\frac{10V}{260\mu V}\right]$ = 92dB

Fig. 2.15. Excerpt from a AD book on the noise of photo-diode converter.

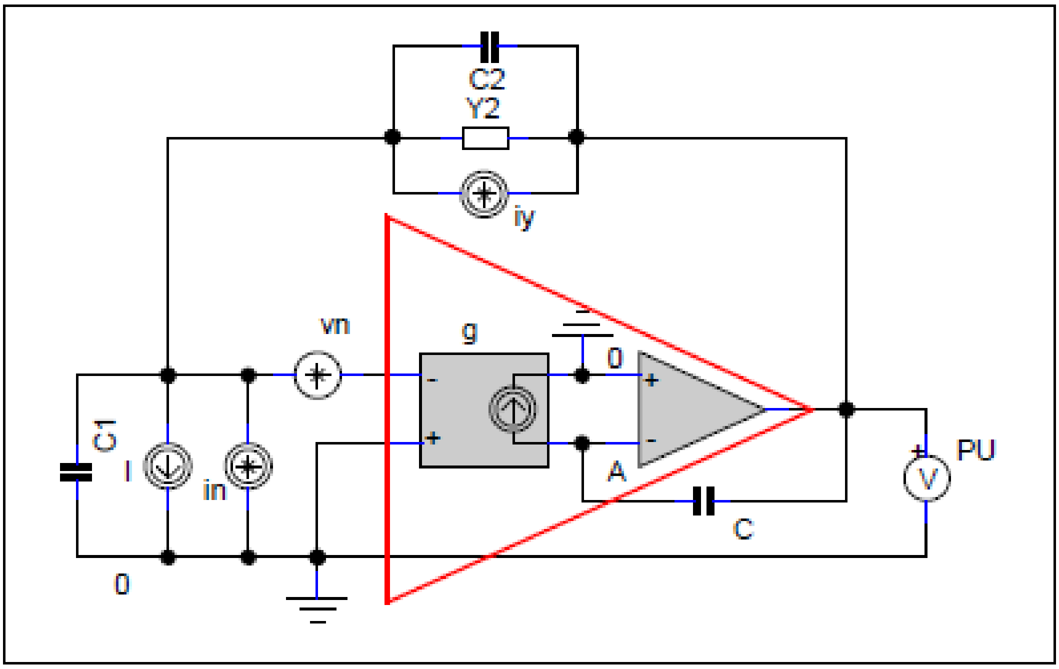


Fig. 2.16. Noise model of photo-diode converter.

Amplifier is modeled as a gain stage with finite gain-bandwidth product $2\pi f_t$=**g/C** (slightly differently than in stability analysis, but phase excess is not so important for noise analysis).
I models a photo-diode current. Amplifier voltage and current noise sources are represent by **vn**, **in**. Feedback resistance noise is represented by **iy.**

```
//...   TF  ...
//  U=Kn*vn+M*I+Mn*in+My*iy
Da= (s^(2)*(C1*C+C2*C)+s*(C*Y2+C2*g)+Y2*g);

DbKn= (g)*(s*(C1+C2)+Y2);
Kn=DbKn/Da;
DbM= (g)*(1);
M=DbM/Da;
DbMn= (g)*(1);
Mn=DbMn/Da;
DbMy= (g)*(1);
My=DbMy/Da;
```

Voltage to current transfer functions are equal for all current sources (which is pretty obvious for **I**,**in** and not for **iy**).
For a given power density of noise sources,

$$S_{en}(f)=e_n^2\cdot(1+\frac{F_{ce}}{f})$$
$$S_{in}(f)=i_n^2\cdot(1+\frac{F_{ci}}{f})$$
$$S_{iy}(f)=4\cdot k\cdot T\cdot Y$$
[2.10]

we use a formula for uncorrelated noise sources to determine $S_o(f)$ - output voltage power noise density:

$$S_o(f)=(S_{in}(f)+S_{iy}(f))\cdot|M_n(f)|^2+S_{en}\cdot|K_n(f)|^2$$
[2.11]

RMS value of noise is found by integration of output noise density for a given bandwidth **f=[f1..f2]**:

$$v_n^2 = \int_{f_1}^{f_2} S_o(f)\, df \qquad [2.8]$$

For parameters from the original text:

```
R2 = 100 [kOhm]
C1 = 5.8 [pF]
C2 = 0.76 [pF]
ft = 16 [MHz]
C = 3 [pF]
g = 0.3 [mS]
e_n = 16 [nV/sqrt(Hz)]
F_ce = 1000 [Hz]
i_n = 0.001 [pA/sqrt(Hz)]
F_ci = 1000 [Hz]
```

[2.9]

Maxima calculates:

```
vo = sqrt(ve^2+vi^2+vy^2) = 233.8 [uV]
vE = 223.2 [uV]
vI = 0.17 [uV]
vY = 69.42 [uV]
```

[2.10]

Value of op-amp voltage noise contribution estimated in fig 2.15 is slightly higher (250uV), but difference is negligible from practical perspective.

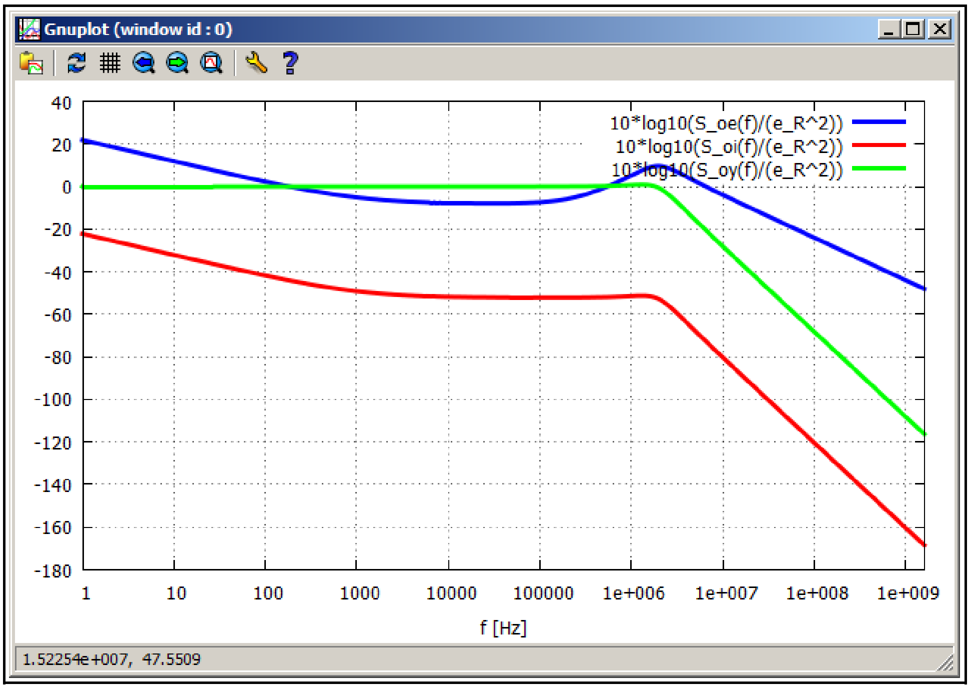


Fig. 2.17. Output power noise density normalized to resistor power noise density e_R^2=4*kT*R

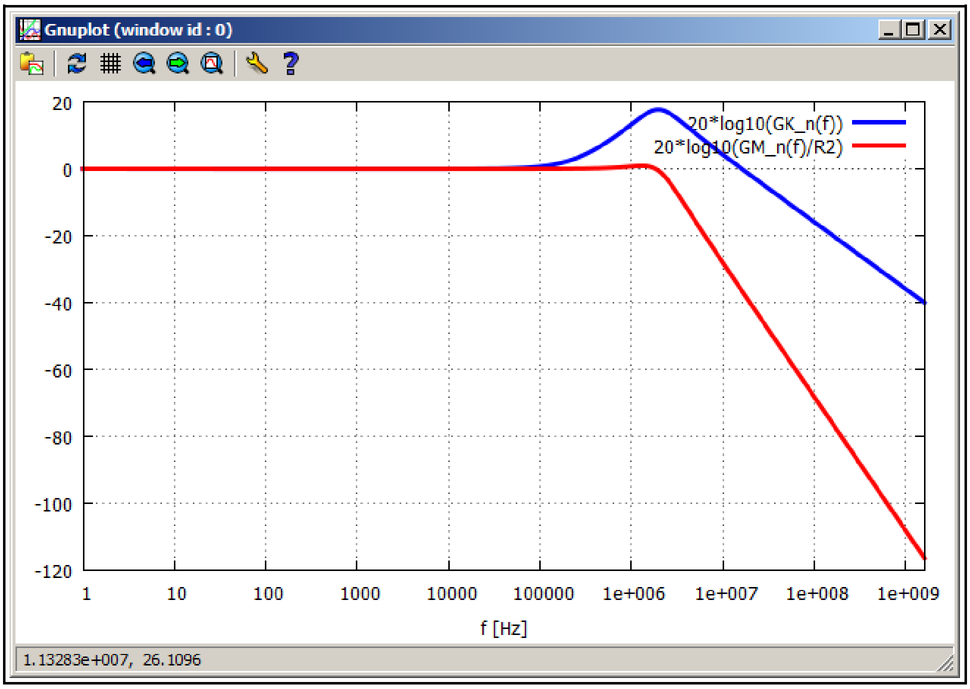


Fig. 2.18. Noise transfer functions

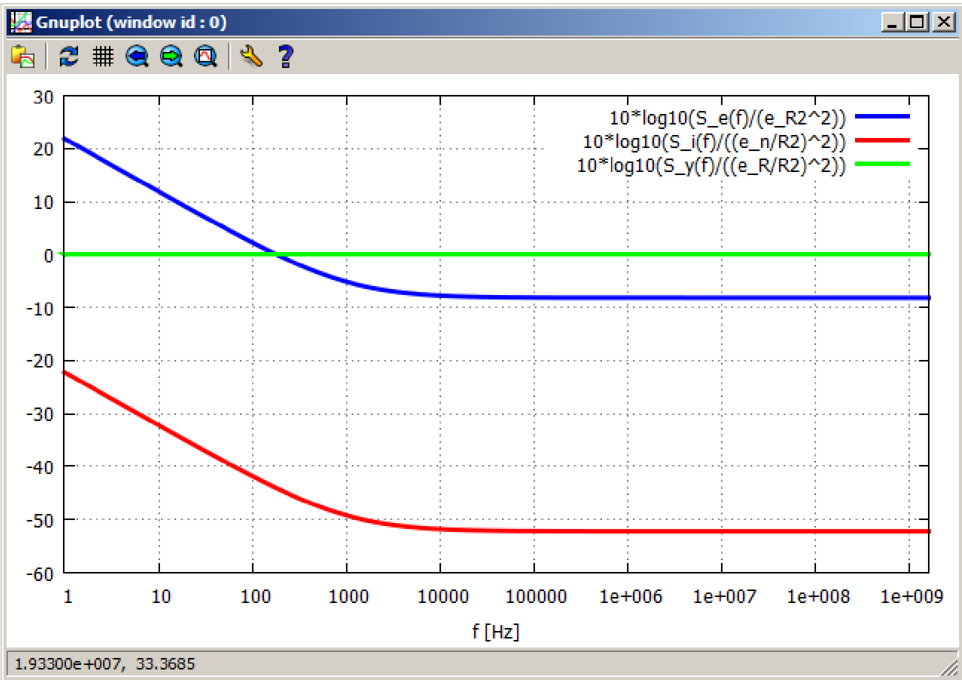


Fig. 2.19. Input power noise density normalized to resistor power noise density e_R^2=4*kT*R

3. References

[1] – http://www.hutson.co.nz/ **- Rimu schematics**.

[2] – http://maxima.sourceforge.net/ **Maxima, a Computer Algebra System.**

[3] - http://www.maximintegrated.com/ REFERENCE SCHEMATIC 5610 - **High-Performance, High-Accuracy 4-20mA Current-Loop Transmitter**

[4] -http://www.analog.com/library/analogdialogue/archives/39-05/Web_Ch4_final.pdf **-Sensors signal conditioning – page 4.53**